

# Adaptive Closed-Loop Identification and Tracking Control of an Aerial Vehicle with Unknown Inertia Parameters <sup>★</sup>

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**Abstract:** In this paper, the problem of adaptive closed-loop parameter estimation and tracking control of a six degree of freedom (6-DOF) nonlinear quadrotor unmanned aerial vehicle (UAV) is studied. To manage the complexity of the problem, the system dynamic is decomposed into two subsystems, i.e. translational dynamics and rotational dynamics. A nested control architecture is adopted to develop both adaptive tracking control and parameter estimation. To stabilise the outer loop a virtual control input is proposed using a proportional-derivative (PD) controller to track the x, y and z position. The rotational dynamics of UAV contains unknown inertia parameters appearing in the control structure as well as in a nonlinear dynamic term. An adaptive tracking scheme is designed using the certainty equivalence principle to handle both parameter estimation and tracking control in closed-loop. The idea behind the controller design is to cancel the nonlinear term in the inner loop by estimating the unknown system parameters. The stability of the whole closed loop system is proved with a rigorous analytical study. Moreover, the performance of the proposed controller is verified with several numerical analyses.

**Keywords:** Grey box identification, 6-DOF quadrotor, certainty equivalence principle, adaptive tracking control, closed-loop identification, unknown inertia parameters.

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## 1. INTRODUCTION

In recent years, the research and development in control of quadrotor unmanned aerial vehicles (UAV) have provided a significant technological advances. The UAV quadrotors have found a broad range of applications such as virtual reality, creative industries (Santamarina-Campos and Segarra-Oña (2018)), monitoring and inspections of hazardous environments (Montazeri et al. (2020); Burrell et al. (2018)), volcano monitoring, and agriculture data collection (Radoglou-Grammatikis et al. (2020)).

One of the topical research directions in the realm of robotics, including the aerial robots, is to improve the autonomy and cognition of robotic systems. This can be viewed as the operation of a single aerial vehicle or a system of aerial vehicles collaborating as a network and as a cyber-physical system (Um (2019); Montazeri et al. (2020)) in a real environment. Therefore, the main objective of the current research is to design a proper controller for the UAV quadrotor to operate in a practical setting.

The presence of nonlinearities in the attitude dynamics of the quadrotor system is one of the most essential issues in designing the controller. The simplest approach

to control a UAV, is to design a linear controller by linearizing the system dynamics around the operating point. This technique has a significant limitation to be applied for smooth maneuvers. This problem is addressed by designing a proper nonlinear controller stabilising the UAV in the full nonlinear operation range of the quad. The first nonlinear technique is the so called feedback linearization technique proposed in (Voos (2009); Zhou et al. (2010)) for trajectory tracking problem of UAVs. This approach only can be implemented if all parameters of quadrotor are available for the feedback control design.

Nevertheless, in many practical scenarios, one or more parameters of UAV may not be available for the feedback control design. As a result, more advance controllers are required. Two common approaches to handle the nonlinearities with uncertain parameters in the UAV dynamic are robust and adaptive controllers. The idea behind robust control is to dominate the uncertain nonlinear term. Some interesting results using robust control can be found in (Huang and Chen (2004); Lewis et al. (2003); Chen (2015)) for the single setting and in (Chen and Chen (2016); Zhu and Chen (2014)) for the collaborative settings. Adaptive control is another approach to handle the nonlinearities with unknown parameters in UAVs. In this approach, the uncertain nonlinear term is maintained by estimating the unknown system parameters. Some results using adaptive control techniques in UAVs can be found in (Narendra and Annaswamy (1989); Anderson et al. (2005); Astolfi et al.

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(2007); Hovakimyan and Cao (2010)) for the single setting and in (Lewis et al. (2013); Imran et al. (2019, 2020)) for the collaborative setting.

In general, two typical scenarios can be imagined where the attitude dynamics has unknown parameters. In the first scenarios the unknown parameters appear in the nonlinear dynamic but they are separated from the control input structure. The results under this setting for the quadrotor application are reported in (Nemati and Montazeri (2018b,a)) using sliding mode control and in (Imran and Montazeri (2020b,a)) using the adaptive control method. In the second scenarios, which is the subject of the current research, the unknown parameters appear in the control input structure. In this case, it is assumed that the inertia parameters of the quadrotor are unknown and as can be seen later they appear in both control input and the nonlinear attitude dynamic.

The remainder of the paper is organized as follows. In section 2 the system dynamics of quadrotor is presented. Following that, the outer loop position tracking control as well as the proposed adaptive tracking attitude control system is presented in Section 3. To demonstrate the performance of designed controller, several numerical simulations are conducted in Section 4. Finally, the paper is concluded in the last section with some recommendations for the future work.

## 2. SYSTEM DYNAMICS OF QUADROTOR

In this section, we present the dynamic model of a 6-DOF quadrotor UAV, including both translational and rotational dynamics. For this purpose, we follow the notations used in (Imran and Montazeri (2020b,a)). The translational dynamics is expressed as

$$\ddot{\eta}_1 = -gz_e + J_1(\eta_2)z_e \frac{u}{m} - \frac{k_t}{m}\dot{\eta}_1, \quad (1)$$

where  $z_e = [0 \ 0 \ 1]^T$ ,  $g$  is the gravity acceleration,  $u$  is the thrust force,  $k_t$  is translational drag coefficient and  $m$  is the mass of UAV. Vector  $\eta_1 = [x \ y \ z]^T$  represents the position vector consisting of forward, lateral, and vertical motions respectively; Here  $\eta_2 = [\phi \ \theta \ \psi]^T$  represents the orientation vector consist of roll ( $\phi$ ), pitch ( $\theta$ ) and yaw ( $\psi$ ) motions. Matrix  $J_1(\eta_2)$  is a transformation matrix expressed by

$$J_1(\eta_2) = \begin{bmatrix} \cos \theta \cos \psi & \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi \\ \cos \theta \sin \psi & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi \\ -\sin \theta & \sin \phi \cos \theta \\ & \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ & \cos \phi \cos \theta \end{bmatrix}.$$

By assuming  $\cos \phi$  and  $\cos \theta$  to be non-zero

$$J_1^T(\eta_2) = J_1^{-1}(\eta_2). \quad (2)$$

The rotational dynamics can be represented as

$$\dot{\nu}_2 = w_1 f(\nu_2) + w_2 \tau, \quad (3)$$

where

$$w_1 = \text{diag} \left[ \frac{I_y - I_z}{I_x}, \frac{I_z - I_x}{I_y}, \frac{I_x - I_y}{I_z} \right]$$

$$f(\nu_2) = [qr \ pr \ pq]^T$$

$$w_2 = I_m^{-1}.$$

The vector  $\nu_2 = [p \ q \ r]^T$  in (3) is the angular velocity vectors,  $I_M = \text{diag}[I_x \ I_y \ I_z]$  is the inertia matrix and  $\tau = [\tau_p \ \tau_q \ \tau_r]^T$  is the torque vector acting on the body frame. Here, it is assumed that the inertia parameters  $I_x$ ,  $I_y$  and  $I_z$  are unknown for the feedback control design.

By putting (1) and (3) together, we get the dynamic model of the UAV as an under-actuated systems, in which the four control inputs are used to control the six system states. Both translational and rotational dynamics are highly coupled between the body and inertial frames and represented by

$$\begin{aligned} \dot{\eta}_1 &= J_1(\eta_2)\nu_1 \\ \dot{\eta}_2 &= J_2(\eta_2)\nu_2, \end{aligned} \quad (4)$$

where  $\nu_1 = [u \ v \ w]^T$  is the linear velocity vector,  $\eta_2 = [\phi \ \theta \ \psi]^T$  is the orientation vector consisting of roll ( $\phi$ ), pitch ( $\theta$ ) and yaw ( $\psi$ ) motions and matrix  $J_2(\eta_2)$  is a transformation matrix represented by

$$J_2(\eta_2) = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & \sin \theta \\ 0 & \frac{\cos \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{bmatrix}.$$

The thrust force  $u$  is generated by

$$u = \sum_{i=1}^4 f_i = \sum_{i=1}^4 k_i \Omega_i^2, \quad (5)$$

where  $f_i$  is upward-lifting force generated by each rotor,  $k_i$  is a positive constant gain and  $\Omega_i$  is the angular speed of the rotor  $i$ . The thrust force and the torques acting around the body of UAV have the following relationship

$$\begin{bmatrix} u \\ \tau_p \\ \tau_q \\ \tau_r \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \frac{l\sqrt{2}}{2} & -\frac{l\sqrt{2}}{2} & -\frac{l\sqrt{2}}{2} & \frac{l\sqrt{2}}{2} \\ \frac{l\sqrt{2}}{2} & \frac{l\sqrt{2}}{2} & -\frac{l\sqrt{2}}{2} & -\frac{l\sqrt{2}}{2} \\ \frac{2}{d} & \frac{2}{-d} & \frac{2}{d} & \frac{2}{-d} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}, \quad (6)$$

where  $l$  is the arm length and  $d$  is the drag factor.

## 3. PROPOSED CONTROL DESIGN

In this section, the proposed controllers for both translational and rotational dynamics are presented. A virtual PD controller is applied to maintain the translational motions of UAV as presented in Section 3.1.

The presence of nonlinearities in the attitude dynamics is an essential issue in designing attitude controller. If all parameters of the dynamics are known for feedback control design, then the control problem can be simplified by applying a fully feedback linearization to cancel the nonlinear terms. However, this is not a viable solution since some parameters are either unavailable or the exact values are unknown for the feedback control design. As a result, a fully feedback linearization approach does not give rise to a good performance. The problem becomes more complicated when the unknown parameters appear in the control input structure. In Section 3.2, an adaptive control technique is developed to handle the tracking control of UAV with inertia parameters  $I_x$ ,  $I_y$  and  $I_z$  as unknown values. These unknown parameters appear in the control input structure as well as in the nonlinear dynamic terms.

### 3.1 Translational control design

In this section, the tracking control for translational dynamics is designed following the results presented in (Imran and Montazeri (2020a)). The tracking error of the system can be defined as

$$\tilde{\eta}_1 = \eta_1 - \eta_{1d}, \quad (7)$$

where  $\tilde{\eta}_1$  and  $\eta_{1d}$  are the error vector position and the desired vector position, respectively. The double integrator dynamics of (7) can be written as

$$\ddot{\tilde{\eta}}_1 = -K_D \dot{\tilde{\eta}}_1 - K_P \tilde{\eta}_1. \quad (8)$$

The control gains  $K_P$  and  $K_D$  are selected to be positive definite, as a result Routh-Hurwitz stability criterion for system dynamics (8) is satisfied. The dynamics (7) can be rewritten as follows

$$\ddot{\eta}_1 = \ddot{\eta}_{1d} - K_D(\dot{\eta}_{1d} - \dot{\eta}_1) - K_P(\eta_{1d} - \eta_1). \quad (9)$$

We define a virtual input  $U = \ddot{\eta}_1 = [U_1 \ U_2 \ U_3]^\top$ . Then by substituting  $U$  to (1), we have

$$U = -gz_e + J_1(\eta_2)z_e \frac{u}{m} - \frac{k_t}{m}\dot{\eta}_1, \quad (10)$$

or

$$\frac{u}{m}z_e = J_1^{-1}(\eta_2)(U + gz_e + \frac{k_t}{m}\dot{\eta}_1). \quad (11)$$

By expanding (11), we have the following relationships

$$\begin{aligned} (U_1 + \frac{k_t}{m}\dot{x}) \cos \theta \cos \psi + (U_2 + \frac{k_t}{m}\dot{y}) \cos \theta \sin \psi \\ - (U_3 + g + \frac{k_t}{m}\dot{z}) \sin \theta = 0, \end{aligned} \quad (12)$$

$$\begin{aligned} (U_1 + \frac{k_t}{m}\dot{x})(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) \\ + (U_2 + \frac{k_t}{m}\dot{y})(\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) \\ + (U_3 + g + \frac{k_t}{m}\dot{z}) \sin \phi \cos \theta = 0, \end{aligned} \quad (13)$$

$$\begin{aligned} (U_1 + \frac{k_t}{m}\dot{x})(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \\ + (U_2 + \frac{k_t}{m}\dot{y})(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\ + (U_3 + g + \frac{k_t}{m}\dot{z}) \cos \phi \cos \theta = \frac{u}{m}. \end{aligned} \quad (14)$$

The fact that  $\cos \theta \neq 0$ . From (12), we can compute  $\theta$  as follows

$$\theta = \arctan \left( \frac{(U_1 + \frac{k_t}{m}\dot{x}) \cos \psi + (U_2 + \frac{k_t}{m}\dot{y}) \sin \psi}{U_3 + g + \frac{k_t}{m}\dot{z}} \right). \quad (15)$$

From (11), we have

$$\begin{aligned} \left( \frac{u}{m}z_e \right)^\top \left( \frac{u}{m}z_e \right) &= \left( J_1^{-1}(\eta_2)(U + gz_e + \frac{k_t}{m}\dot{\eta}_1) \right)^\top \\ &\quad \left( J_1^{-1}(\eta_2)(U + gz_e + \frac{k_t}{m}\dot{\eta}_1) \right) \\ &= \left( U + gz_e + \frac{k_t}{m}\dot{\eta} \right)^\top \left( U + gz_e \right. \\ &\quad \left. + \frac{k_t}{m}\dot{\eta}_1 \right). \end{aligned} \quad (16)$$

Therefore

$$\begin{aligned} \frac{u}{m} &= \left( (U_1 + \frac{k_t}{m}\dot{x})^2 + (U_2 + \frac{k_t}{m}\dot{y})^2 \right. \\ &\quad \left. + (U_3 + g + \frac{k_t}{m}\dot{z})^2 \right)^{1/2} \end{aligned} \quad (17)$$

From (13) and (14), we obtain

$$\frac{u}{m} \sin(\phi) = (U_1 + \frac{k_t}{m}\dot{x}) \sin(\psi) - (U_2 + \frac{k_t}{m}\dot{y}) \cos(\psi) \quad (18)$$

We can compute the  $\phi$  by substituting (17) to (18) as

$$\begin{aligned} \phi &= \arcsin \left( \left( (U_1 + \frac{k_t}{m}\dot{x}) \sin \psi - (U_2 + \frac{k_t}{m}\dot{y}) \cos \psi \right) \right. \\ &\quad \left. \left( (U_1 + \frac{k_t}{m}\dot{x})^2 + (U_2 + \frac{k_t}{m}\dot{y})^2 \right. \right. \\ &\quad \left. \left. + (U_3 + g + \frac{k_t}{m}\dot{z})^2 \right)^{1/2} \right). \end{aligned} \quad (19)$$

By following similar arguments, thus we can generate  $\phi_d$  and  $\theta_d$  as expressed by

$$\begin{aligned} \phi_d &= \arcsin \left( \left( (U_1 + \frac{k_t}{m}\dot{x}_d) \sin \psi_d - (U_2 + \frac{k_t}{m}\dot{y}_d) \right. \right. \\ &\quad \left. \left. \cos \psi_d \right) \left( (U_1 + \frac{k_t}{m}\dot{x}_d)^2 + (U_2 + \frac{k_t}{m}\dot{y}_d)^2 \right. \right. \\ &\quad \left. \left. + (U_3 + g + \frac{k_t}{m}\dot{z}_d)^2 \right)^{1/2} \right) \end{aligned} \quad (20)$$

$$\theta_d = \arctan \left( \frac{(U_1 + \frac{k_t}{m}\dot{x}_d) \cos \psi_d + (U_2 + \frac{k_t}{m}\dot{y}_d) \sin \psi_d}{U_3 + g + \frac{k_t}{m}\dot{z}_d} \right). \quad (21)$$

The total thrust  $u$  is generated from (14), as expressed by

$$\begin{aligned} u &= m \left( (U_1 + \frac{k_t}{m}\dot{x})(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \right. \\ &\quad + (U_2 + \frac{k_t}{m}\dot{y})(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \\ &\quad \left. + (U_3 + g + \frac{k_t}{m}\dot{z}) \cos \phi \cos \theta \right). \end{aligned} \quad (22)$$

### 3.2 Attitude control design

In this section, an adaptive scheme for attitude dynamics of UAV with unknown inertia parameters is developed. The main objective of the adaptive controller is to stabilise the closed loop system so that not only the tracking error is going to zero but also to estimate the inertia parameters of the system simultaneously. By defining the desired trajectory  $\nu_{2d} = [p_d \ q_d \ r_d]^\top$  and the tracking error as  $e = \nu_2 - \nu_{2d}$ , the tracking error dynamics can be written as

$$\dot{e} = -\dot{\nu}_{2d} + w_1 f(\nu_2) + w_2 \tau. \quad (23)$$

The tracking controller is deemed to be successful if

$$\lim_{t \rightarrow \infty} e(t) = 0. \quad (24)$$

Before presenting the main results, we define

$$\begin{aligned} E &= \text{diag}(e), \\ F(\nu_2) &= \text{diag}(f(\nu_2)), \\ N_{2d} &= \text{diag}(\nu_{2d}). \end{aligned}$$

The main result of the proposed controller for attitude dynamics is presented in Theorem 3.1.

**Theorem 3.1.** Consider the attitude dynamics (3). The objective of the tracking error (24) is achieved by selecting the controller

$$\tau = -\hat{w}_2 (\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2d}), \quad (25)$$

with adaptation law

$$\begin{aligned} \dot{\hat{w}}_1 &= \gamma_1 F(\nu_2) E, \\ \dot{\hat{w}}_2 &= \gamma_2 E (\alpha E + \hat{w}_1 F(\nu_2) - N_{2d}), \end{aligned} \quad (26)$$

where  $\alpha$ ,  $\gamma_1$  and  $\gamma_2$  are some positive constants.

*Proof:* The dynamics error of closed-loop system (3) under control input (25) can be calculated as follows

$$\begin{aligned} \dot{e} &= -\dot{\nu}_{2d} + w_1 f(\nu_2) - w_2 (\hat{w}_2 (\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2d})) \\ &= -\dot{\nu}_{2d} + w_1 f(\nu_2) - w_2 (\tilde{w}_2 + w_2^{-1}) (\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2d}) \\ &= -\dot{\nu}_{2d} + w_1 f(\nu_2) - w_2 \tilde{w}_2 (\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2d}) \\ &\quad - \alpha e - \hat{w}_1 f(\nu_2) + \dot{\nu}_{2d} \\ &= -\alpha e - \tilde{w}_1 f(\nu_2) - w_2 \tilde{w}_2 (\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2d}), \end{aligned} \quad (27)$$

where  $\tilde{w}_1 = \hat{w}_1 - w_1$  and  $\tilde{w}_2 = \hat{w}_2 - w_2^{-1}$ .

We select the Lyapunov function of dynamics (23) to be

$$V(e, \tilde{w}_1, \tilde{w}_2) = \frac{1}{2} e^T e + \text{tr} \left( \frac{1}{2\gamma_1} \tilde{w}_1^2 + \frac{1}{2\gamma_2} w_2 \tilde{w}_2^2 \right). \quad (28)$$

Direct calculation shows that the time-derivative of  $V(e, \tilde{w}_1, \tilde{w}_2)$  along the closed-loop system (3)+(25)+(26) is

$$\begin{aligned} \dot{V}(e, \tilde{w}_1, \tilde{w}_2) &= e^T \dot{e} + \text{tr} \left( \frac{1}{\gamma_1} \tilde{w}_1 \dot{\tilde{w}}_1 + \frac{1}{\gamma_2} w_2 \tilde{w}_2 \dot{\tilde{w}}_2 \right) \\ &= e^T \left( -\alpha e - \tilde{w}_1 f(\nu_2) - w_2 \tilde{w}_2 (\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2d}) \right) + \text{tr} \left( \frac{1}{\gamma_1} \tilde{w}_1 \dot{\tilde{w}}_1 + \frac{1}{\gamma_2} w_2 \tilde{w}_2 \dot{\tilde{w}}_2 \right) \\ &= -\alpha e^T e + e^T \left( -\tilde{w}_1 f(\nu_2) - w_2 \tilde{w}_2 (\alpha e + \hat{w}_1 f(\nu_2) - \dot{\nu}_{2d}) \right) \\ &\quad + \text{tr} \left( \frac{1}{\gamma_1} \tilde{w}_1 \dot{\tilde{w}}_1 + \frac{1}{\gamma_2} w_2 \tilde{w}_2 \dot{\tilde{w}}_2 \right) \\ &= -\alpha e^T e + \text{tr} \left( \frac{1}{\gamma_1} \tilde{w}_1 \dot{\tilde{w}}_1 + \frac{1}{\gamma_2} w_2 \tilde{w}_2 \dot{\tilde{w}}_2 - \tilde{w}_1 F(\nu_2) E - w_2 \tilde{w}_2 E (\alpha E + \hat{w}_1 F(\nu_2) - N_{2d}) \right) \\ &= -\alpha e^T e. \end{aligned}$$

From (26) and (27), we can see that  $e(t)$ ,  $\tilde{w}_1$  and  $\tilde{w}_2$  are bounded. To show the tracking error  $e$  is driven asymptotically to zero, we calculate the second time-derivative of Lyapunov function  $V(e, \tilde{w}_1, \tilde{w}_2)$  as

$$\ddot{V}(e, \tilde{w}_1, \tilde{w}_2) = -2\alpha e^T \dot{e}. \quad (29)$$

It shows from (27) that  $e$  is uniformly bounded, and hence  $\ddot{V}(e, \tilde{w}_1, \tilde{w}_2)$  is bounded. This implies that  $\dot{V}(e, \tilde{w}_1, \tilde{w}_2)$  is uniformly continuous. By Barbalat's Lemma, then  $\lim_{t \rightarrow \infty} e(t) = 0$ . Therefore, the proof is completed. ■

## 4. SIMULATION RESULTS

The performance of the proposed approach is evaluated numerically in this section. The parameters of the quadrotor UAV used in this simulation is presented in Table 1

Table 1. The parameters of a quadrotor UAV

Parameter name	Notation	Value
Mass	$m$	$0.52kg$
Gravity acceleration	$g$	$9.8m/s^2$
Translational drag coefficient	$k_t$	$0.95$
Arm length	$l$	$0.205m$
Drag factor	$d$	$7.5e^{-7}kg.m^2g$
Inertia of $x$ -axis	$I_x$	$0.0069kg.m^2$
Inertia of $y$ -axis	$I_y$	$0.0069kg.m^2$
Inertia of $z$ -axis	$I_z$	$0.0129kg.m^2$

To maintain the translational motions, the simulation is conducted using a virtual PD controller (8) with  $K_P = K_D = \text{diag}([100 \ 100 \ 100])$ . In another side, the Theorem 3.1 is proposed to maintain the attitude motions. The gains of (25) and (26) are selected to be

$$\alpha = 10000, \gamma_1 = 0.1, \gamma_2 = 0.01.$$

The simulation results for tracking control of UAV for both the translational and rotational motions are illustrated in Figures 1-4. We can verify that all states of UAV can follow the desired trajectories, as concluded in Theorem 3.1. Moreover, we also present the performance of adaptation law estimation error as presented in Figure 5 and 6. The profiles of torque  $\tau$  and UAV motions in three dimension (3D) can be seen in Figure 7 and 8, respectively.

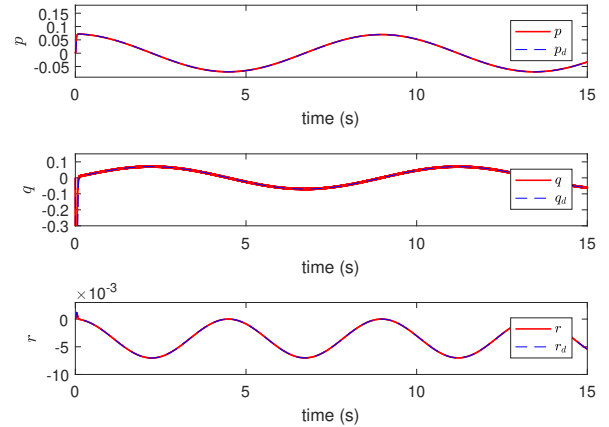


Fig. 1. Profile of  $p$ ,  $q$  and  $r$

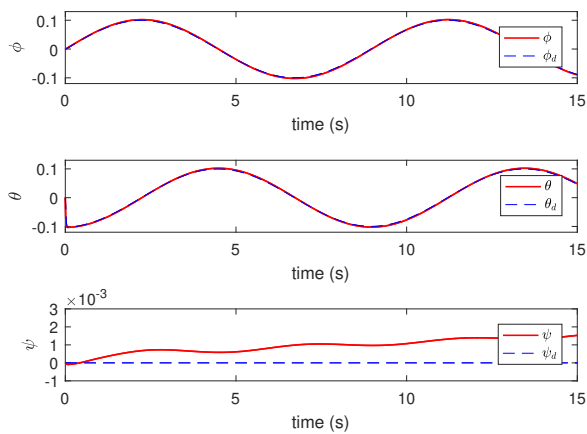


Fig. 2. Profile of  $\phi$ ,  $\theta$  and  $\psi$

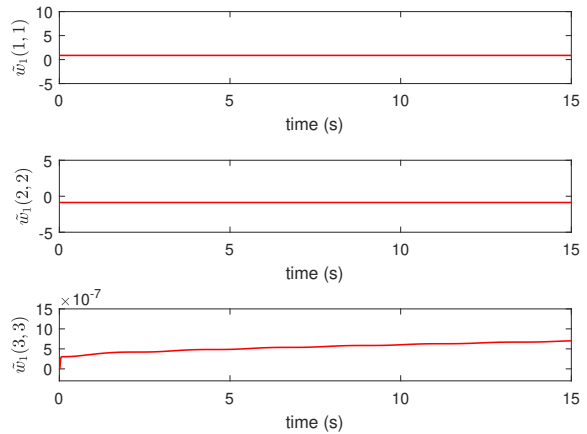


Fig. 5. Profile of estimation error  $\tilde{w}_1$

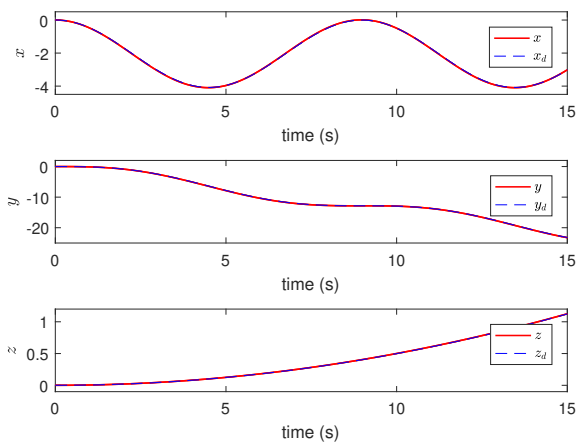


Fig. 3. Profile of  $x$ ,  $y$  and  $z$

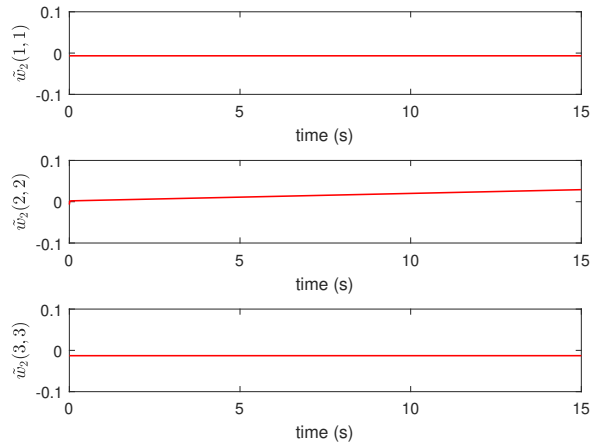


Fig. 6. Profile of estimation error  $\tilde{w}_2$

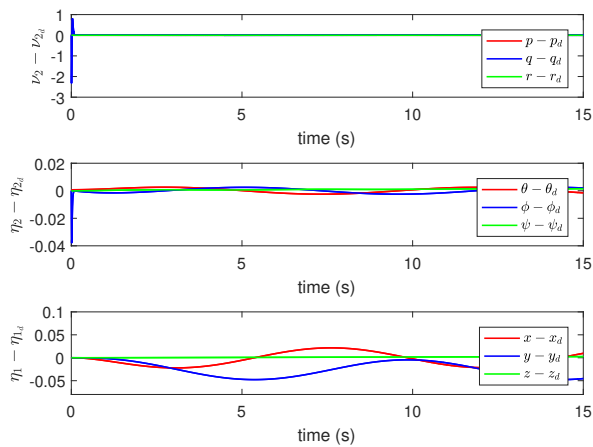


Fig. 4. Profile of tracking error trajectories

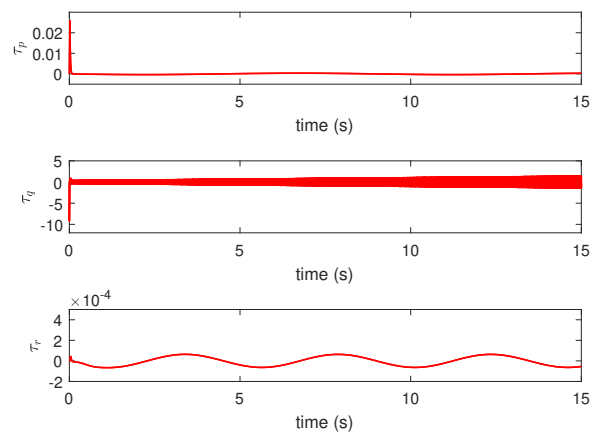


Fig. 7. Profile of  $\tau$

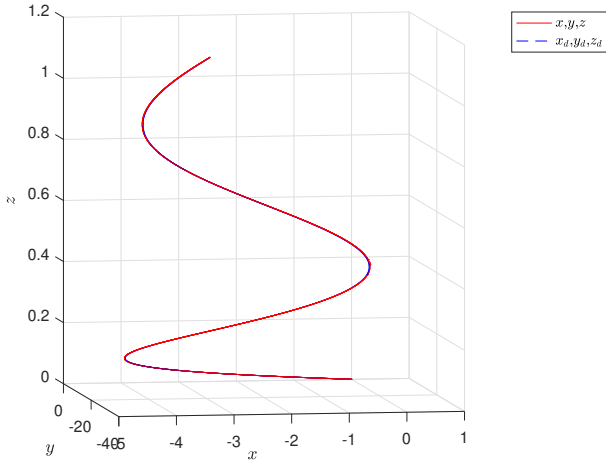


Fig. 8. Profile of  $x$ ,  $y$  and  $z$

## 5. CONCLUSION

A full 6-DOF tracking control of UAV with unknown inertia parameters is presented in this paper. A virtual PD controller is proposed for the tracking position control in the outer loop. The main contribution is to design a closed-loop parameter estimation and adaptive controller for rotational dynamics in the presence of unknown inertia parameters in both the control input and the nonlinear dynamic of the system. The effectiveness of the tracking controller is proven with a rigorous proof using the Barbalat's Lemma. To demonstrate the performance of the proposed approach, a numerical study is conducted for the mini-quadrotor. It will be interesting to apply the scheme in real scenarios as the future work.

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## REFERENCES

- Anderson, B.D. et al. (2005). Failures of adaptive control theory and their resolution. *Communications in Information & Systems*, 5(1), 1–20.
- Astolfi, A., Karagiannis, D., and Ortega, R. (2007). *Nonlinear and adaptive control with applications*. Springer Science & Business Media.
- Burrell, T., West, C., Monk, S.D., Montazeri, A., and Taylor, C.J. (2018). Towards a cooperative robotic system for autonomous pipe cutting in nuclear decommissioning. In *2018 UKACC 12th International Conference on Control (CONTROL)*, 283–288. IEEE.
- Chen, X. and Chen, Z. (2016). Robust perturbed output regulation and synchronization of nonlinear heterogeneous multiagents. *IEEE Trans. Cybernetics*, 46(12), 3111–3122.
- Chen, Z. (2015). A novel adaptive control approach for nonlinearly parameterized systems. *International Journal of Adaptive Control and Signal Processing*, 29(1), 81–98.
- Hovakimyan, N. and Cao, C. (2010). *L1 adaptive control theory: guaranteed robustness with fast adaptation*, volume 21. SIAM-Society for Industrial and Applied Mathematics.
- Huang, J. and Chen, Z. (2004). A general framework for tackling the output regulation problem. *IEEE Transactions on Automatic Control*, 49(12), 2203–2218.
- Imran, I.H., Chen, Z., Yan, Y., and Fu, M. (2019). Adaptive consensus of nonlinearly parameterized multi-agent systems. *IEEE Control Systems Letters*, 3(3), 505–510.
- Imran, I.H., Chen, Z., Zhu, L., and Fu, M. (2020). A distributed adaptive scheme for multiagent systems. *Asian Journal of Control*.
- Imran, I.H. and Montazeri, A. (2020a). Adaptive control of quadrotor unmanned aerial vehicle with uncertainties. *IEEE Access (Submitted)*.
- Imran, I.H. and Montazeri, A. (2020b). An adaptive scheme to estimate unknown parameters of an unmanned aerial vehicle. In *2020 International Conference on Nonlinearity, Information and Robotics*.
- Lewis, F.L., Dawson, D.M., and Abdallah, C.T. (2003). *Robot manipulator control: theory and practice*. CRC Press.
- Lewis, F.L., Zhang, H., Hengster-Movric, K., and Das, A. (2013). *Cooperative control of multi-agent systems: optimal and adaptive design approaches*. Springer Science & Business Media.
- Montazeri, A., Can, A., and Imran, I.H. (2020). Unmanned aerial systems: Autonomy, cognition and control. In *Unmanned aerial systems: theoretical foundation and applications*. Elsevier.
- Narendra, K.S. and Annaswamy, A.M. (1989). *Stable adaptive systems*. Prentice Hall: Englewood Cliffs, NJ.
- Nemati, H. and Montazeri, A. (2018a). Analysis and design of a multi-channel time-varying sliding mode controller and its application in unmanned aerial vehicles. *IFAC-PapersOnLine*, 51(22), 244–249.
- Nemati, H. and Montazeri, A. (2018b). Design and development of a novel controller for robust attitude stabilisation of an unmanned air vehicle for nuclear environments. In *2018 UKACC 12th International Conference on Control (CONTROL)*, 373–378. IEEE.
- Radoglou-Grammatikis, P., Sarigiannidis, P., Lagkas, T., and Moscholios, I. (2020). A compilation of uav applications for precision agriculture. *Computer Networks*, 172, 107148.
- Santamarina-Campos, V. and Segarra-Oña, M. (2018). *Drones and the Creative Industry: Innovative Strategies for European SMEs*. Springer Nature.
- Um, J.S. (2019). *Drones as Cyber-Physical Systems*. Springer.
- Voos, H. (2009). Nonlinear control of a quadrotor micro-uav using feedback-linearization. In *2009 IEEE International Conference on Mechatronics*, 1–6. IEEE.
- Zhou, Q.L., Zhang, Y., Rabbath, C.A., and Theilliol, D. (2010). Design of feedback linearization control and reconfigurable control allocation with application to a quadrotor uav. In *2010 Conference on Control and Fault-Tolerant Systems (SysTol)*, 371–376. IEEE.
- Zhu, L. and Chen, Z. (2014). Robust homogenization and consensus of nonlinear multi-agent systems. *Systems & Control Letters*, 65, 50–55.